

项目名称：非线性分数阶微分系统定性理论的若干研究

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项目简介：近年来，来自粘弹性力学、机器人控制以及航空动力学等诸多领域的分数阶微分模型的求解，迫切需要加强分数阶微分系统定性理论与数值方法的研究。该项目主要致力于研究非线性分数阶微分系统的定性理论及其数值解。通过应用非线性泛函分析工具，完成了对几类分数阶非线性(脉冲)微分系统解的性态的研究，在所研系统的(正)解、多个解、最大最小解、唯一解以及一致收敛于解的迭代逼近序列和近似解的误差估计式上取得重大突破。该项目处于国际前沿水平，获得了一系列研究结果，研究成果以学术论文的形式提交展现，共发表 SCI 论文 41 篇，其中 SCI 二区以上论文十多篇，11 篇发表在 TOP 期刊上，7 篇论文被入选“ESI”顶级论文和高引论文。

Monotone iterative technique for boundary value problems of a nonlinear fractional differential equation with deviating arguments

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ABSTRACT

By using the method of upper and lower solutions and the monotone iterative technique, we investigate boundary value problems for fractional differential equations with nonlinear boundary conditions and deviating arguments. As an application, an example is presented to illustrate the main results.

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1. Introduction

Differential equations of fractional order occur more frequently in different research areas and engineering, such as physics, chemistry, aerodynamics, electro-dynamics of complex medium, polymer rheology, control of dynamical systems, etc. It is well known that fractional derivatives are generalizations of derivatives of integer order, and there are several kinds of fractional derivatives, such as the Riemann–Liouville fractional derivative, Marchaud fractional derivative, Caputo fractional derivative, etc. Since as cited in [1,2], there have appeared a number of works, especially in the theory of viscoelasticity and in hereditary solid mechanics, where fractional derivatives are used to do a better description of material properties.

In this paper, we shall study the following boundary value problems for fractional differential equations with nonlinear boundary conditions and deviating arguments

$$\begin{cases} D^q u(t) = f(t, u(t), u(\tau(t))), & t \in J = (0, T) \quad (1.1) \\ u(0) = u(T) = 0, \end{cases}$$

where $f \in C(J, T \times R \times R, R)$, $g \in C(R \times R, R)$, $\tau \in C(J, J)$, $u(0) = \tau^{-1} \circ u(\tau(t))$, $u(T) = \tau^{-1} \circ u(\tau(t))$, and $D^q u$ is the Riemann–Liouville fractional derivative of u and τ is such that $0 < \tau < 1$.

Differential equations with deviating arguments appear often in investigations connected with mathematical physics, mechanics, engineering, economics and so on (see [1–5]). Recently, many people have paid more and more attention to the existence of a solution to nonlinear differential equations, such as [6–11], and have gained a series of results, for example, some basic theory for initial value problems of fractional differential equations involving the Riemann–Liouville differential operator was discussed in [16–21]. However, the theory of boundary value problems for nonlinear fractional differential equations is still in the initial stages. The recent surge in developing the theory of fractional differential equations has motivated the present work.

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(1)

Impulsive anti-periodic boundary value problem for nonlinear differential equations of fractional order

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ABSTRACT

In this paper, we prove the existence and uniqueness of solutions for an anti-periodic boundary value problem of nonlinear impulsive differential equations of fractional order $\alpha \in (2, 3)$ by applying some well-known fixed point theorems. Some examples are presented to illustrate the main results.

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1. Introduction

Fractional differential equations have recently been addressed by several researchers for a variety of problems. The interest in the study of differential equations of fractional order lies in the fact that fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. With this advantage, the fractional-order models become more realistic and practical than the classical integer-order models, in which such effects are not taken into account. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. [1–4]. For some recent development on the topic, see [5–18] and the references therein.

Anti-periodic problems constitute an important class of boundary value problems and have recently received considerable attention. Anti-periodic boundary conditions occur in the mathematical modeling of a variety of physical processes; for instance see [19–20] and the references therein. The recent results on anti-periodic boundary value problems of fractional differential equations can be found in [20–22].

The theory of impulsive differential equations of integer order has found its extensive applications in realistic mathematical modeling of a wide variety of practical situations and has emerged as an important area of investigation in recent years. For the general theory and applications of impulsive differential equations, we refer the reader to Refs. [23–28]. However, impulsive differential equations of fractional order have not been much studied and many aspects of these equations are yet to be explored. For some recent work on impulsive fractional differential equations, see [29–48] and the references therein.

Motivated by some recent work on anti-periodic and impulsive boundary value problems of fractional order, we investigate the existence and uniqueness of solutions for an anti-periodic boundary value problem of nonlinear impulsive

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(2)

图 1 ★ 该成果构建了分数阶比较原理，通过上下解方法和单调迭代技巧，获得了一类分数阶微分方程极值解的存在性，被入选“ESI”顶级论文和高引论文。

图 2 ★ 该成果建立了一类非线性分数阶脉冲微分方程反周期边值问题解的存在性理论，被入选“ESI”顶级论文和高引论文。

Existence results and the monotone iterative technique for systems of nonlinear fractional differential equations^a

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ABSTRACT

By establishing a comparison result and using the monotone iterative technique combined with the method of upper and lower solutions, we investigate the existence of solutions for systems of nonlinear fractional differential equations.

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1. Introduction

Fractional differential equations arise in many engineering and scientific disciplines as the mathematical modeling of systems and processes in the fields of physics, chemistry, aerodynamics, electro-dynamics of complex medium, polymer rheology, etc. (see [1–4] and references therein). Since, as cited in [2], a number of works have appeared, especially in the theory of viscoelasticity and in hereditary solid mechanics, where fractional derivatives are used to do a better description of material properties, some basic theory for fractional differential equations involving the Riemann–Liouville fractional operator has been discussed by many authors [5–16]. On the other hand, the study of systems involving fractional differential equations is also important as such systems occur in various problems of applied nature, for example, see [17–22].

In this paper, we discuss some existence results for systems of nonlinear fractional differential equations. In order to obtain the solutions of systems of nonlinear fractional differential equations, we also develop the monotone iterative technique. It is well known that the method of upper and lower solutions coupled with its associated monotone iteration scheme is an interesting and powerful mechanism that offers theoretical as well constructive existence results for nonlinear problems in a closed set, generated by the lower and upper solutions. For instance, see [23,27]. To the best of our knowledge, this technique has not been applied yet to the systems of nonlinear fractional differential equations.

Consider the following system of nonlinear fractional differential equations

$$\begin{cases} D^q u(t) = f(t, u(t), v(t)), & t \in (0, T) \\ D^q v(t) = g(t, u(t), v(t)), & t \in (0, T) \\ u^+(0) = u^-(0) = u_0, & v^+(0) = v^-(0) = v_0, \end{cases} \quad (1.1)$$

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(3)

Nonlinear fractional integro-differential equations on unbounded domains in a Banach space

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ABSTRACT

In this paper, by employing the fixed point theory and the monotone iterative technique, we investigate the existence of solutions of nonlinear fractional integro-differential equations on semi-infinite domains in a Banach space. An explicit iterative sequence for approximating the solution of the boundary value problem is derived. An error estimate is also given.

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1. Introduction

The extensive application of fractional calculus in the mathematical modeling of physical, engineering and biological phenomena has motivated several researchers to explore theoretical as well as practical aspects of the subject. It is learnt through experimentation that the integral and derivative operators of fractional order do share some of the characteristics exhibited by the processes associated with complex systems having long-memory in time. Thus fractional models are the natural substitutes of the classical integer-order model for such systems. Fractional calculus also provides an excellent tool to describe the hereditary properties of various materials and processes. Concerning the development of theory, methods and applications of fractional calculus, we refer the books [1–5]. Some recent results on fractional differential equations with finite domains, for instance, can be found in papers [6–14] and the references cited therein. Though much of the work on fractional calculus deals with finite domain yet there is a considerable development on the topic involving unbounded domain [15–23].

In this paper, we investigate the existence of solutions for a fractional nonlinear integro-differential equation of mixed type on a semi-infinite interval in a Banach space E:

$$\begin{cases} D^q u(t) = f(t, u(t), \mathcal{I}_\alpha u(t)), & t = 1 \leq t < \infty, \alpha \in \mathbb{N}, \alpha \geq 2, \\ u(0) = u'(0) = u''(0) = \dots = u^{(\alpha-2)}(0) = 0, & D^q u(\infty) = u_\infty. \end{cases} \quad (1.1)$$

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(4)

图 3 ★ 该成果开创了研究非线性分数阶微分方程组的上下解方法和单调迭代技巧，被入选“ESI”顶级论文和高引论文。

图 4 ★ 该成果建立了无界域上一类分数阶非线性积分-微分方程的单

调迭代理论，被入选“ESI”顶级论文和高引论文。

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SUCCESSIVE ITERATIONS FOR POSITIVE EXTREMAL
SOLUTIONS OF NONLINEAR FRACTIONAL
DIFFERENTIAL EQUATIONS ON A HALF-LINE
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Abstract

In this paper, positive solutions of fractional differential equations with nonlinear terms depending on lower-order derivatives on a half-line are investigated. The positive extremal solutions and iterative schemes for approximating them are obtained by applying a monotone iterative method. An example is presented to illustrate the main results.

2010 *Mathematics subject classification*: primary 34A08; secondary 34B10, 34B18.

Keywords and phrases: monotone iterative method, fractional differential equation, positive extremal solutions, half-line.

1. Introduction

Fractional calculus has gained considerable attention from both theoretical and applied points of view in recent years. There are numerous applications in a variety of fields such as electrical networks, chemical physics, fluid flow, economics, signal and image processing, viscoelasticity, porous media, aerodynamics, modelling for physical phenomena exhibiting anomalous diffusion, and so on. In contrast to integer-order differential and integral operators, fractional-order differential operators are nonlocal in nature and provide the means to look into hereditary properties of several materials and processes. This aspect of fractional-order operators has helped to improve the mathematical modelling of many real-world problems in the physical and technical sciences. A detailed description of theory and applications of the subject can be found in the texts [3, 8, 9, 17, 19].

Another important contribution of fractional calculus has been observed in the investigation of backward problems. It is well known that the backward problem in time is *severely* ill-posed for the parabolic problem (involving a first-order derivative

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(5)

图 5★ 该成果确立了一类非线性项依赖低阶导数的分数阶微分方程正极大值解的存在性和迭代逼近序列，被入选“ESI”顶级论文和高引论文。