项目名称：非线性分数阶微分系统定性理论的若干研究

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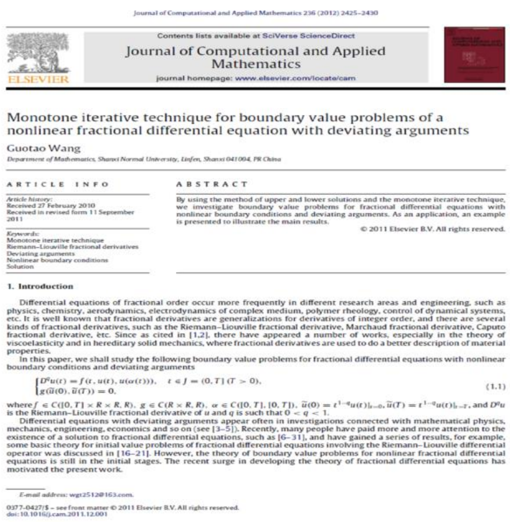
获奖奖种及等级:山西省科学技术奖(自然科学类)三等奖

项目简介：近年来，来自粘弹性力学、机器人控制以及航空动力学等诸多领域的分数阶微分模型的求解，迫切需要加强分数阶微分系统定性理论与数值方法的研究。该项目主要致力于研究非线性分数阶微分系统的定性理论及其数值解。通过应用非线性泛函分析工具，完成了对几类分数阶非线性（脉冲）微分系统解的性态的研究，在所研系统的（正）解、多个解、最大最小解、唯一解以及一致收敛于解的迭代逼近序列和近似解的误差估计式上取得重大突破．该项目处于国际前沿水平，获得了一系列研究结果，研究成果以学术论文的形式提交展现，共发表 SCI 论文 41 篇，其中 SCI 二区以上论文十多篇，11 篇发表在 TOP 期刊上，7 篇论文被入选“ESI”顶级论文和高引论文。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Monotone iterative technique for boundary value problems of a nonlinear fractional differential equation with deviating arguments

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ABSTRACT

By using the method of upper and lower solutions and the monotone iterative technique, we investigate boundary value problems for fractional differential equations with nonlinear boundary conditions and deviating arguments. As an application, an example is presented to illustrate the main results.

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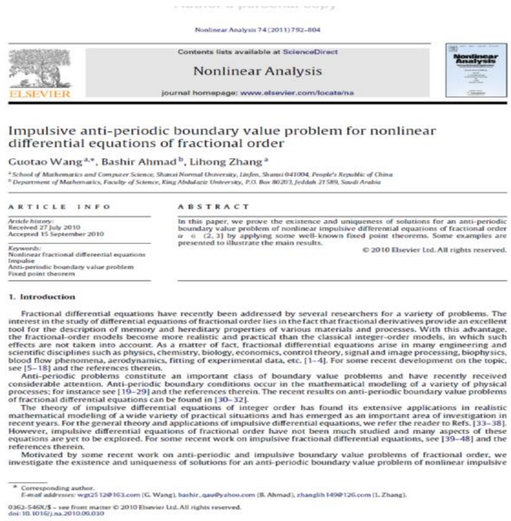
1. Introduction

Differential equations of fractional order occur more frequently in different research areas and engineering, such as physics, chemistry, aerodynamics, electrodynamics of complex medium, polymer rheology, control of dynamical systems, etc. It is well known that fractional derivatives are generalizations for derivatives of integer order, and there are several kinds of fractional derivatives, such as the Riemann–Liouville fractional derivative, Marchaud fractional derivative, Caputo fractional derivative, etc. Since as cited in [1,2], there have appeared a number of works, especially in the theory of viscoelasticity and in hereditary solid mechanics, where fractional derivatives are used to do a better description of material properties.

In this paper, we shall study the following boundary value problems for fractional differential equations with nonlinear boundary conditions and deviating arguments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Impulsive anti-periodic boundary value problem for nonlinear differential equations of fractional order

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ABSTRACT

In this paper, we prove the existence and uniqueness of solutions for an anti-periodic boundary value problem of nonlinear impulsive differential equations of fractional order $\alpha \in (2, 3]$ by applying some well-known fixed point theorems. Some examples are presented to illustrate the main results.

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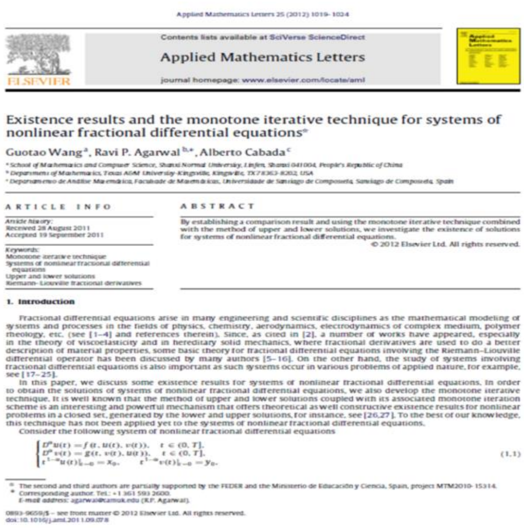
1. Introduction

Fractional differential equations have recently been addressed by several researchers for a variety of problems. The interest in the study of differential equations of fractional order lies in the fact that fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. With this advantage, the fractional-order models become more realistic and practical than the classical integer-order models, in which such effects are not taken into account. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. [1–4]. For some recent development on the topic, see [5–18] and the references therein.

(1) (2)

图 1★ 该成果构建了分数阶比较原理，通过上下解方法和单调迭代技巧，获得了一类分数阶微分方程极值解的存在性，被入选“ESI”顶级论文和高引论文。

图 2★ 该成果建立了一类非线性分数阶脉冲微分方程反周期边值问题解的存在性理论，被入选“ESI”顶级论文和高引论文。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Existence results and the monotone iterative technique for systems of nonlinear fractional differential equations*

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ABSTRACT

By establishing a comparison result and using the monotone iterative technique combined with the method of upper and lower solutions, we investigate the existence of solutions for systems of nonlinear fractional differential equations.

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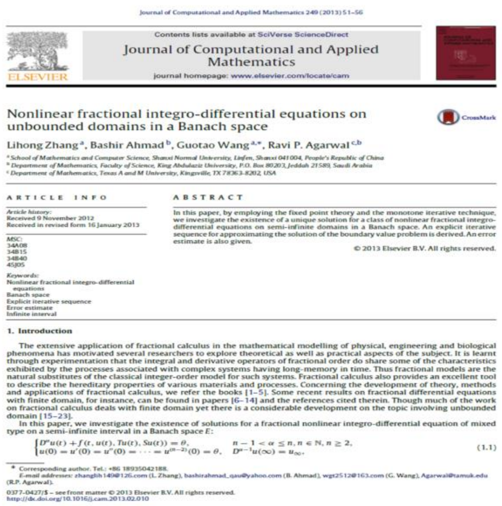
1. Introduction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Nonlinear fractional integro-differential equations on unbounded domains in a Banach space

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ABSTRACT

In this paper, by employing the fixed point theory and the monotone iterative technique, we investigate the existence of a unique solution for a class of nonlinear fractional integro-differential equations on semi-infinite domains in a Banach space. An explicit iterative sequence for approximating the solution of the boundary value problem is derived. An error estimate is also given.

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1. Introduction

The extensive application of fractional calculus in the mathematical modelling of physical, engineering and biological phenomena has motivated several researchers to explore theoretical as well as practical aspects of the subject. It is learnt through experimentation that the integral and derivative operators of fractional order do share some of the characteristics

(3) (4)

图 3★ 该成果开创了研究非线性分数阶微分方程组的上下解方法和单调迭代技巧，被入选“ESI”顶级论文和高引论文。

图 4★ 该成果建立了无界域上一类分数阶非线性积-微分方程的单

调迭代理论，被入选“ESI”顶级论文和高引论文。

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SUCCESSIVE ITERATIONS FOR POSITIVE EXTREMAL SOLUTIONS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS ON A HALF-LINE

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Abstract

In this paper, positive solutions of fractional differential equations with nonlinear terms depending on lower-order derivatives on a half-line are investigated. The positive extremal solutions and iterative schemes for approximating them are obtained by applying a monotone iterative method. An example is presented to illustrate the main results.

2010 *Mathematics subject classification*: primary 34A08; secondary 34B10, 34B18.

Keywords and phrases: monotone iterative method, fractional differential equation, positive extremal solutions, half-line.

1. Introduction

Fractional calculus has gained considerable attention from both theoretical and applied points of view in recent years. There are numerous applications in a variety of fields such as electrical networks, chemical physics, fluid flow, economics, signal and image processing, viscoelasticity, porous media, aerodynamics, modelling for physical phenomena exhibiting anomalous diffusion, and so on. In contrast to integer-order differential and integral operators, fractional-order differential operators are nonlocal in nature and provide the means to look into hereditary properties of several materials and processes. This aspect of fractional-order operators has helped to improve the mathematical modelling of many real-world problems in the physical and technical sciences. A detailed description of theory and applications of the subject can be found in the texts [3, 8, 9, 17, 19].

Another important contribution of fractional calculus has been observed in the investigation of backward problems. It is well known that the backward problem in time is *severely* ill-posed for the parabolic problem (involving a first-order derivative

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(5)

图 5★ 该成果确立了一类非线性项依赖低阶导数的分数阶微分方程正极值解的存在性和迭代逼近序列，被入选“ESI”顶级论文和高引论文。